Protocols and Outer Bounds for the Gaussian Two-way Diamond Channel

A Project Report

submitted by

PRATHYUSHA V

in partial fulfilment of the requirements
for the award of the degree of

MASTER OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY MADRAS

MAY 2013

THESIS CERTIFICATE

This is to certify that the thesis titled **Protocols and Outer Bounds for the Gaussian Two-way Diamond Channel**, submitted by **Prathyusha V**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work done by her under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Prof. Srikrishna Bhashyam
Project Guide
Associate Professor
Dept. of Electrical Engineering
IIT-Madras, 600 036

Place: Chennai

Date: 16/05/2013

ACKNOWLEDGEMENTS

First and foremost, I would like to express my sincere thanks to my project guide Dr. Srikrishna Bhashyam. I am highly grateful to him for the guidance, encouragement and freedom he has given me throughout the course of this project. I would also like to thank Dr. Andrew Thangaraj for his help and encouragement during this project.

I would like to thank the Department of Electrical Engineering for providing an excellent laboratory and computational resources for making this project possible. I would like to acknowledge all the professors of the department whose courses strengthened my concepts and deepened my interest in various topics. I would also like to thank my faculty advisor Dr. David Koilpillai for his guidance over the past two years.

I would also like to thank all my labmates - Maria, Hussam, Vinay, Kiran, Archana, D Kiran, Sreekanth and others for their help and encouragement in my work and for making the lab so enjoyable throughout the year. I also thank all my friends - Renuka, Sanjukta, Ekta, Aseem, Asit, Aneesh, Aparna, Kruthika and others for making my two years of stay at IIT Madras such a wholesome experience. Special thanks are due to my batch-mates of M.Tech Communication who have been wonderful company, tough competitors and great friends.

Finally, I would like to thank my family for all the support and encouragement in making my IIT dream possible. I dedicate my work to them.

ABSTRACT

The Gaussian two-way relay channel and the Gaussian two-way diamond channel are studied. First, we derive an analytical expression for the outer bound to the capacity region of the two-way relay channel. This analytical outer bound is found to achieve symmetric capacity within 0.5 bits for some channel conditions where the direct link is weak. A Gaussian two-way diamond channel consists of two nodes communicating with each other with the help of two half-duplex relays in diamond topology. Using multiple relays increases capacity of the network. We consider mixing of the two flows at the relays to increase the capacity further. This is accomplished using physical layer network coding or the compute and forward strategy. We propose two protocols for the diamond channel based on this scheme - the CF-CMAC and the CF-BC protocols. We also extend the uni-directional Multi-hopping Decode and Forward(MDF) protocol to the case of two-way relaying over a diamond channel, by time sharing between the flows in the two directions. We observe that the compute and forward based schemes achieve several rate pairs that cannot be achieved by simple time sharing between the two one way flows. Finally, we propose an outer bound to the achievable rate region of the diamond channel using the cut-set bound. We observe that the proposed protocols achieve rates close to the outer bound under some channel conditions. We also derive outer bounds to the capacity region of the gaussian diamond channel with direct link between relays and with direct link between the nodes. The outer bounds are compared to investigate the effect of direct links on the achievable rate region.

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ABBREVIATIONS

AF	Amplify and Forward
CF	Compute and Forward
DF	Decode and Forward
MF	Mixed Forward
LF	Lattice Forward
CF-CMAC	Compute and Forward - Compound Multiple Access Channel
CF-BC	Compute and Forward - Broadcast Channel
MDF	Multi-hopping Decode and Forward

NOTATION

A,B	Terminal Nodes
R, R_1, R_2	Relay Nodes
h	Channel Gain
$\mathcal{C}$	Capacity
γ	Signal-to-noise ratio
N	Receiver Noise Variance
P	Transmit Power of each node
R_a	Data transmission rate from node A to node B
R_b	Data transmission rate from node B to node A
λ_i, μ_i	Fraction of channel use in state i
Z_{ij}^k	Information flow from node i to node j in state k
Λ	Lattice
ν	Fundamental Voronoi Region of the lattice

CHAPTER 1

INTRODUCTION

The relay channel was first introduced by van der Muelen in 1971. A two-way or bidirectional relay channel consists of two nodes exchanging messages through one or more relays. Relay networks find applications in multi-hop wireless networks, sensor networks with transmitter power limitations etc.. Relay networks of different topologies have been studied under different relaying schemes like Amplify and Forward (AF), Decode and Forward (DF), Compress and Forward (CF) and Lattice forward. Among the different relay channels, the three node two-way relay channel and the diamond relay channel have attracted significant interest. We restrict our attention to these two channels in the current thesis. We consider half-duplex relays since they are more practical and cost-efficient than full-duplex relays.

Three node two-way relaying with and without direct link have been studied in [1, 2, 3, 4, 5] and [6, 7, 8, 9]. In [6, 1, 2], the achievable rate regions of various two-way relaying protocols are compared. A generalised outer bound for the capacity region for all relaying protocols was derived in [10] using the half-duplex cutset bound. This outer bound considers all possible states of the network, as opposed to the bounds in [1, 2, 3] which are protocol specific. In the present thesis, we derive an analytical outer bound for capacity of the three node two-way channel(as opposed to an optimisation problem formulation). Using this analytical outer bound, we obtain the symmetric capacity within 0.5 bits for some channel conditions where the direct link is weak.

The half-duplex Gaussian diamond relay channel where a source node and a destination node communicate with each other through two non-interfering relays has been studied in [11], [12] and [13]. In [12], the the AF, CF and DF relaying schemes have been generalised to the diamond channel and some hybrid schemes also have been proposed. In [13], multi-hopping decode-and-forward (MDF) protocols for one-way communication were proposed to achieve rates within a constant gap of a capacity outer bound. The diamond channel with interfering relays was studied in [14]. In the present thesis, we consider the problem of two-way communication over a diamond relay channel, which appears to have not received attention in existing literature. We propose relaying protocols for the bidirectional communication over this channel, and determine their achievable rate regions. An interesting aspect of two-way relaying is that there are two flows and and mixing of the two flows at the relay can be exploited to improve the rates in both directions. This is achieved by physical layer network coding or the compute-and-forward strategy. Coding schemes based on nested lattice coding and compute and forward have been proposed in [15, 16]. In this work, we apply the nested lattice coding scheme in [15] to the case of diamond relay channel. Based on this, we propose two protocols- The CF-CMAC(Compute and Forward Multiple Access Channel) and the CF-BC(Compute and Forward-Broadcast), that use compute and forward scheme at the two relays. The CF-CMAC is a three state protocol and uses nested lattice codes. The two terminal nodes simultaneously transmit to one relay in the first state and to the second relay in the second state. The third state is a Compound MAC in which the two relays simultaneously broadcast to both nodes. In the CF-BC protocol, the first two states are similar to the CF-CMAC, followed by two states in which one relay broadcasts to the two nodes at a time. The use of Compound MAC state in CF-CMAC increases the achievable rate region significantly over the CF-BC. We also extend the MDF protocol in [13] to two-way relaying by simple time sharing between the flows in the two directions. We call this the two-way MDF protocol. The first four states constitute flow in one direction, and four more states constitute flow in the reverse direction. We also analyse and compare the achievable rate regions of all the three protocols. Finally, we look into the possibility of time sharing between these protocols to increase the achievable rate region. Protocols for the diamond channel with direct links are also proposed by extending the approaches in [3] and [14].

We also derive outer bounds to the capacity regions of the gaussian diamond channel with and without direct links. This is done by extending the approach in [10] to the diamond channel. The outer bounds are compared to investigate the effect of the direct links on capacity. The outer bounds for the capacity region of the diamond channel are compared with the achievable rate regions of the proposed protocols.

1.1 Organization of Thesis

The organization of this thesis is as follows.

- Chapter 2 discusses the generalised numerical outer bound for the capacity of the Gaussian three node two-way relay channel and derives an analytical outer bound for the same.
- Chapter 3 discusses the Gaussian Two-way Diamond channel without direct link and some of the existing protocols for this channel.
- Chapter 4 discusses some new protocols for the Gaussian Two-way Diamond channel - Two-way MDF, CF-BC and CF-CMAC protocols.
- Chapter 5 deals with the derivation of outer bound to the capacity region of the gaussian diamond relay channel without direct links.
- Chapter 6 deals with the achievable rates and outer bounds of the gaussian diamond channel with direct links.
- Chapter 7 presents some numerical results for the achievable rate regions of different protocols and comparison with outer bounds.
- Chapter 8 concludes this thesis discussing possible future works.

CHAPTER 2

THE GAUSSIAN TWO-WAY RELAY CHANNEL

2.1 System Model

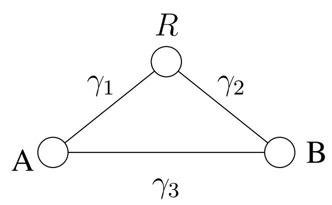

Figure 2.1: Two-way Gaussian Relay Channel with Direct Link

Consider a two-way relay channel with 3 nodes - two terminal nodes A and B wanting to communicate with each other and relay R assisting communication between them. All nodes are half duplex. The network can be in different states depending on whether each of the nodes is in transmit or receive state. The channel is Gaussian with a receiver noise variance of N. Let P be the transmit power of all nodes. The channel is assumed to be reciprocal and constant. Channel state information is assumed to be available at all nodes. The SNRs of the different links are denoted as $\gamma_1 = \frac{h^2{}_1P}{N}$,$\gamma_2 = \frac{h^2{}_2P}{N}$ and $\gamma_3 = \frac{h^2{}_3P}{N}$, where h_1, h_2, h_3 are the gains of the links $a \leftrightarrow r$, $b \leftrightarrow r$, $a \leftrightarrow b$ respectively. We use $C(\gamma) = log_2(1+\gamma)$ to represent the capacity of a complex gaussian channel with SNR γ.

Several protocols have been proposed for two-way communication over the Gaussian relay channel and their achievable rate regions have been obtained. Protocol specific outer bounds for the capacity region have also been derived and the gap between the achievable rate region and the outer bound has been characterised. A generalised outer bound for the capacity of the half-duplex two way relay channel was proposed in [10], which is applicable to all protocols. This outer bound is proposed as a solution of a linear program. In this thesis, we derive an analytical expression that is also an outer bound by analysing the dual of the linear program in [10]. This can be used to gain a better understanding of the bound.

2.2 Analytical Outer Bound for the Capacity Region of the Gaussian Two-Way Relay Channel

The outer bound for the capacity region of the Gaussian Two-way relay channel has been obtained in [10] as:
Given $R_a = kR_b$ for some $k \geq 0$, the maximum possible R_b is upper-bounded by C_{bk} obtained by solving the following linear program:

$$C_{bk} = \max_{R_b, \{\lambda_i\}} R_b,$$

subject to

$$\begin{aligned} kR_b &\leq \lambda_1 \mathcal{C}(\gamma_1 + \gamma_3) + \lambda_3 \mathcal{C}(\gamma_1) + \lambda_5 \mathcal{C}\left(\gamma_3\right), \\ kR_b &\leq \lambda_1 \mathcal{C}(\gamma_3) + \lambda_4 \mathcal{C}(\gamma_2) + \lambda_5 \mathcal{C}\left((\sqrt{\gamma_2} + \sqrt{\gamma_3})^2\right), \\ R_b &\leq \lambda_2 \mathcal{C}(\gamma_2 + \gamma_3) + \lambda_3 \mathcal{C}(\gamma_2) + \lambda_6 \mathcal{C}\left(\gamma_3\right), \\ R_b &\leq \lambda_2 \mathcal{C}(\gamma_3) + \lambda_4 \mathcal{C}(\gamma_1) + \lambda_6 \mathcal{C}\left((\sqrt{\gamma_1} + \sqrt{\gamma_3})^2\right), \\ \sum_{i=1}^{6} &\lambda_i \leq 1,\ \lambda_i > 0,\ R_b > 0. \end{aligned} \tag{2.1}$$

The dual of the above linear program can be written as:

$$\min_{\{y_i\}} y_5$$

subject to

$$\begin{aligned} y_5 &\geq y_1 \mathcal{C}(\gamma_1 + \gamma_3) + y_2 \mathcal{C}\left(\gamma_3\right), \\ y_5 &\geq y_3 \mathcal{C}(\gamma_2 + \gamma_3) + y_4 \mathcal{C}\left(\gamma_3\right), \\ y_5 &\geq y_1 \mathcal{C}(\gamma_1) + y_3 \mathcal{C}\left(\gamma_2\right), \\ y_5 &\geq y_2 \mathcal{C}(\gamma_2) + y_4 \mathcal{C}\left(\gamma_1\right), \\ y_5 &\geq y_1 \mathcal{C}(\gamma_3) + y_2 \mathcal{C}\left((\sqrt{\gamma_2} + \sqrt{\gamma_3})^2\right), \\ y_5 &\geq y_3 \mathcal{C}(\gamma_3) + y_4 \mathcal{C}\left((\sqrt{\gamma_1} + \sqrt{\gamma_3})^2\right), \\ ky_1 &+ ky_2 + y_3 + y_4 \geq 1. \end{aligned} \tag{2.2}$$

The value of the dual program at any feasible point is an upper bound on the value of the primal problem. Choosing an appropriate feasible point in the dual program provides a good upper bound on R_b. The bound on R_b is obtained by choosing:

$$y_1 = \frac{2k-1}{2k^2}\frac{\mathcal{C}(\gamma_2)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}; \qquad y_2 = \frac{2k-1}{2k^2}\frac{\mathcal{C}(\gamma_1)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)};$$

$$y_3 = \frac{1}{2k}\frac{\mathcal{C}(\gamma_1)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}; \qquad y_4 = \frac{1}{2k}\frac{\mathcal{C}(\gamma_2)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}$$

For any $k \geq 1$, R_b is upper-bounded as

$$R_b \leq \max\{T_1, T_2, T_3, T_4\},$$

where

$$T_1 = \frac{3k-1}{2k^2}\frac{\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)},$$

$$T_2 = \frac{2k-1}{2k^2}\frac{\mathcal{C}(\gamma_2)\mathcal{C}(\gamma_1+\gamma_3)+\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_3)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)},$$

$$T_3 = \frac{2k-1}{2k^2}\frac{\mathcal{C}(\gamma_2)\mathcal{C}(\gamma_3)+\mathcal{C}(\gamma_1)\mathcal{C}((\sqrt{\gamma_2}+\sqrt{\gamma_3})^2)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)},$$

$$T_4 = \frac{1}{2k}\frac{\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_3)+\mathcal{C}(\gamma_2)\mathcal{C}((\sqrt{\gamma_1}+\sqrt{\gamma_3})^2)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}.$$

This analytical outer bound can be used to simplify the characterisation of capacity for different channel conditions. As an illustration, for $k=1$ case, $T_2 \leq T_4$. Therefore, we get $R_b \leq \max\{T_1, T_3, T_4\}$. For $\gamma_1 = \gamma_2 = \gamma$ and $k=1$, we get

$$R_b \leq \max\left\{\frac{\mathcal{C}(\gamma)}{2}, \frac{1}{4}\left[\mathcal{C}(\gamma_3)+\mathcal{C}((\sqrt{\gamma}+\sqrt{\gamma_3})^2)\right]\right\}. \tag{2.3}$$

A similar result can be obtained for $k < 1$ as well. In this case, we can set $R_b = k'R_a$, where $k' > 1$ and use the same technique as above. The expressions obtained for T_1 to T_4 will be similar except that γ_1 and γ_2 are interchanged in each expression.

R_b is also upper bounded by the upper bound for one-way relaying from B to A with $R_a = 0$, i.e., we have

$$R_b \leq \frac{\mathcal{C}(\gamma_1+\gamma_3)\mathcal{C}((\sqrt{\gamma_2}+\sqrt{\gamma_3})^2)-\mathcal{C}^2(\gamma_3)}{\mathcal{C}(\gamma_1+\gamma_3)+\mathcal{C}((\sqrt{\gamma_2}+\sqrt{\gamma_3})^2)-2\mathcal{C}(\gamma_3)}.$$

This bound is obtained by solving the dual program for one-way relaying. When there is no direct link, i.e., when $\gamma_3 = 0$, this reduces to

$$R_b \leq \frac{\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)}{\mathcal{C}(\gamma_1) + \mathcal{C}(\gamma_2)},$$

where the bound is the capacity of the half-duplex two-hop linear network [17].

Capacity Results:

We can obtain the following results for capacity from (2.3).

1. For $\gamma_1 = \gamma_2 = \gamma$ and $k = 1$, the upper bound on R_a is $\mathcal{C}(\gamma)/2$ for $\gamma_3 \leq \gamma_{30}$, where γ_{30} satisfies $f(\gamma_{30}) = 2\mathcal{C}(\gamma)$ where $f(\gamma_3) \triangleq \mathcal{C}(\gamma_3) + \mathcal{C}((\sqrt{\gamma} + \sqrt{\gamma_3})^2)$. That means, when the direct link is weak, the upper bound for capacity is $\mathcal{C}(\gamma)/2$. The lattice coding scheme in [15] can achieve rates within 0.5 bits of this upper bound without any direct link. Therefore, for the weak direct link regime specified by γ_{30}, $\mathcal{C}(\gamma)/2$ is the capacity within 0.5 bits and can be achieved without using the direct link.

2. For $\gamma_1 \neq \gamma_2$ and $k = 1$, the upper bound on R_a is $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)/(\mathcal{C}(\gamma_1) + \mathcal{C}(\gamma_2))$ for $\gamma_3 \leq \min(\gamma_{31}, \gamma_{32})$, where γ_{31}, γ_{32} satisfy $f_1(\gamma_{31}) = f_2(\gamma_{32}) = 2\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)$ where $f_1(\gamma_3) \triangleq \mathcal{C}(\gamma_2)\mathcal{C}(\gamma_3) + \mathcal{C}(\gamma_1)\mathcal{C}((\sqrt{\gamma_2} + \sqrt{\gamma_3})^2)$ and $f_2(\gamma_3) \triangleq \mathcal{C}(\gamma_1)\mathcal{C}(\gamma_3) + \mathcal{C}(\gamma_2)\mathcal{C}((\sqrt{\gamma_1} + \sqrt{\gamma_3})^2)$.This reduces to $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)/(\mathcal{C}(\gamma_1) + \mathcal{C}(\gamma_2))$ when the direct link is weak. This is equal to the one-way relaying bound when $\gamma_1 \neq \gamma_2$ without the direct link.

CHAPTER 3

THE GAUSSIAN TWO-WAY DIAMOND CHANNEL WITHOUT DIRECT LINKS

3.1 System Model

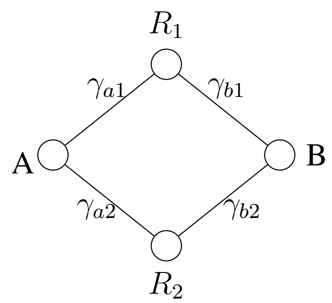

Figure 3.1: Gaussian Diamond Channel

Consider two nodes A and B whose bi-directional communication is assisted by two relays, R_1 and R_2 in diamond topology. No link is present between the two nodes A and B or between the two relays. So all communication is through the two non-interfering relays. All nodes are half duplex. The network can be in different states depending on whether each of the nodes is in transmit or receive state. The channel is Gaussian with a receiver noise variance of N. The channel is assumed to be known, reciprocal and constant. Channel state information is assumed to be available at all nodes. Let P be the transmit power of all nodes. The SNRs of the different links are denoted as $\gamma_{a1} = \frac{h^2{}_{a1}P}{N}$, $\gamma_{a2} = \frac{h^2{}_{a2}P}{N}$, $\gamma_{b1} = \frac{h^2{}_{b1}P}{N}$ and $\gamma_{b2} = \frac{h^2{}_{b2}P}{N}$, where N is the receiver noise variance. We use $C(\gamma) = log_2(1+\gamma)$ to represent the capacity of a complex gaussian channel with SNR γ.

The diamond relay network has 16 possible states, because each of the four nodes can be either in transmit or receive state(half-duplex constraint). So, on the whole there are 2^4 = 16 states. Of these, we can ignore the 2 states in which all nodes are in transmit or all are in receive states, as they do not serve any purpose. The remaining 14 useful states can be seen in Figure 3.2. For example, in state 3, nodes A and R_2 are transmitting to nodes R_1 and B respectively which are receiving.

Let R_a denote the rate of communication from A to B and R_b denote the rate of communication from B to A.

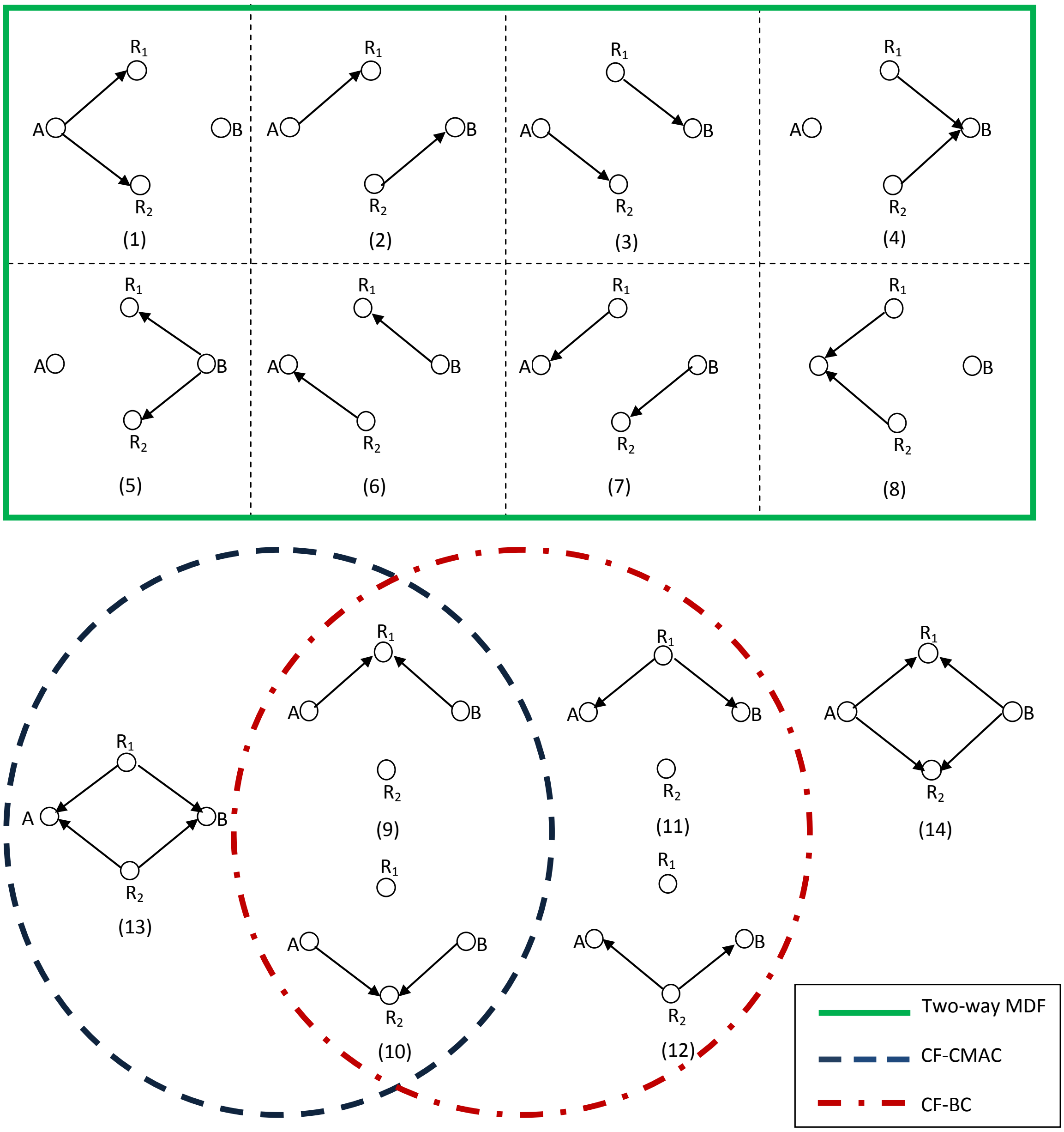

Figure 3.2: States in a half-duplex Gaussian Diamond Relay Channel

3.2 Existing protocols for a Gaussian Diamond Relay channel

The problem of two-way communication over a diamond relay channel that we deal with in this thesis has not been given attention in existing literature. But protocols have been proposed for one-way communication over a diamond channel. We discuss some of them in this section.

Multi-hop with spatial reuse: Multi-hop with spatial reuse[12] is a decode and forward protocol using states 2 and 3 of Figure 3.2. In this scheme, in the first state, A sends a new message to R_1 which decodes it. In the same state, R_2 sends the message previously received from A, to B. In State 2, A sends new message to R_2, which decodes, and R_1 sends the message previously received from A to B. This scheme makes full spatial usage of the network. Denoting the end to end rate through R_1 as Z_1 and through R_2 as Z_2, the maximum achievable rate using this protocol is

$$\begin{aligned} C_{MS} = \max_{\lambda, Z_1, Z_2} & \; Z_1 + Z_2 \text{ s.t} \\ Z_1 &\leq \lambda \mathcal{C}(\gamma_{a1}), \\ Z_2 &\leq \lambda \mathcal{C}(\gamma_{b2}), \\ Z_1 &\leq (1-\lambda) \mathcal{C}(\gamma_{b1}), \\ Z_2 &\leq (1-\lambda) \mathcal{C}(\gamma_{a2}), \\ 0 &\leq \lambda \leq 1, \\ Z_1 &\geq 0, Z_2 \geq 0. \end{aligned} \tag{3.1}$$

Broadcast-Multiaccess With Common Message: There are two states- Broadcast and Multiple Access. In the first state, A sends two independent information streams by broadcast, the two relays. The relay with the better channel to A can decode both messages, so the message to the relay with a weaker link is the 'common' message. In the next state, the two relays send the two information streams to the destination by using multiple access with common message. If R_1 transmits its own message with rate Z_1 and the common message with rate Z_2, and R_2 sends only the common message, then the capacity region is characterized as

$$
\begin{aligned}
C_{BM} &= \max_{\lambda, Z_1, Z_2} Z_1 + Z_2 \text{ s.t} \\
Z_1 &\leq \lambda \mathcal{C}(\alpha \gamma_{a1}), \\
Z_2 &\leq \lambda \mathcal{C}(\frac{(1-\alpha)\gamma_{a2}}{1+\alpha\gamma_{a2}}), \\
Z_1 &\leq (1-\lambda)\mathcal{C}((1-\rho)\gamma_{b1}), \\
Z_1 + Z_2 &\leq (1-\lambda)\mathcal{C}(\gamma_{b1} + \gamma_{b2} + 2\sqrt{\rho\gamma_{b1}\gamma_{b2}}), \\
0 &\leq \lambda \leq 1, Z_1 \geq 0, Z_2 \geq 0.
\end{aligned}
\tag{3.2}
$$

Multi-hop Decode and Forward protocol: The multi-hop decode and forward(MDF) protocol was proposed in [13] for one-way relaying. It is a four state protocol that uses states 1-4 of Figure 3.2. Relaying scheme used is decode and forward, messages are decoded at the end of each state. This scheme is shown to achieve the capacity of the diamond channel when $C(\gamma_{a1})C(\gamma_{a2}) = C(\gamma_{b1})C(\gamma_{b2})$. The achievable rate region for this protocol can be written as

State 1:

$$
\begin{aligned}
Z^1_{ar_1} &\leq \lambda_1 \mathcal{C}(\alpha\gamma_{a1}), \\
Z^1_{ar_2} &\leq \lambda_1 \mathcal{C}\left(\frac{(1-\alpha)\gamma_{a2}}{1+\alpha\gamma_{a2}}\right).
\end{aligned}
\tag{3.3}
$$

State 2:

$$
\begin{aligned}
Z^2_{ar_1} &\leq \lambda_2 \mathcal{C}(\gamma_{a1}), \\
Z^2_{r_2 b} &\leq \lambda_2 \mathcal{C}(\gamma_{b2}).
\end{aligned}
\tag{3.4}
$$

State 3:

$$
\begin{aligned}
Z^3_{ar_2} &\leq \lambda_3 \mathcal{C}(\gamma_{a2}), \\
Z^3_{r_1 b} &\leq \lambda_3 \mathcal{C}(\gamma_{b1}).
\end{aligned}
\tag{3.5}
$$

State 4:

$$
\begin{aligned}
Z_{r_1b}^4 &\leq \lambda_4 \mathcal{C}(\gamma_{b1}), \\
Z_{r_2b}^4 &\leq \lambda_4 \mathcal{C}(\gamma_{b2}), \\
Z_{r_1b}^4 + Z_{r_2b}^4 &\leq \lambda_4 \mathcal{C}(\gamma_{b1} + \gamma_{b2}).
\end{aligned} \tag{3.6}
$$

CHAPTER 4

PROTOCOLS FOR THE TWO-WAY GAUSSIAN DIAMOND CHANNEL

This chapter discusses some new protocols for two-way communication over the Gaussian Diamond Relay channel - The Two-way MDF protocol, the CF-CMAC protocol and the CF-BC protocol. Specifying a relaying protocol involves specifying the sequence of states used and the coding/decoding schemes for each of these states.

4.1 Two-way MDF protocol

Two-way MDF is a simple two-way protocol that uses the MDF protocol of [13] for both direction flows ($A \to B$ and $B \to A$) in a time-sharing manner. Thus, States 1-4 in Figure 3.2 will be used for communication from A to B, and States 5-8 will be used for communication from B to A. We name this the Two-way MDF protocol. The two-way MDF uses Decode and Forward scheme at the relays.

Consider the flow from A to B. State 1 is a broadcast state in which A transmits independent messages to both the relays and the relays decode these messages at the end of the state. In state 2, A sends a new message to R_1 and R_2 transmits a re-encoded version of the message received in state 1 to B. In state 3, A sends a new message to R_2 while R_1 re-encodes and transmits the messages decoded from states 1 and 2 to B. The residual information is forwarded to B by the two relays in State 4 which is a MAC state.

The maximum achievable rate for the one-way MDF protocol can be computed as in [13]. Suppose this rate for communication from A to B is R_{a-mdf} and for communication from B to A is R_{b-mdf}, then the achievable rate region for the two-way MDF protocol is the triangular region enclosed by the three straight lines: (1) $R_a = 0$, (2) $R_b = 0$, and (3) the line joining $(0, R_{b-mdf})$ and $(R_{a-mdf}, 0)$.

4.2 CF-CMAC Protocol

The Compute-and-forward-Compound MAC (CF-CMAC) protocol is a three state protocol using States 9, 10 and 13 of Figure 3.2. States 9 and 10 are Multiple Access Channels in which both the nodes A and B transmit to one of the relays. State 13 is an interference channel in which both the relays simultaneously transmit to both nodes A and B. States 9 and 10 employ nested lattice coding scheme as in [18]. This protocol employs a compute and forward strategy at the relays, making use of the fact that a relay need not decode all the information received by it, it only needs to forward sufficient information to enable the receiving nodes to decode correctly. The relays attempt to decode the sum of the messages received from A and B, instead of decoding the individual messages. This sum is forwarded to the two nodes in State 13.

Encoding and Decoding Using Doubly Nested Lattice codes: In [19], Erez and Zamir showed that nested lattice codes can be used to approach the capacity of point-to-point AWGN channels. A nested lattice code $\mathcal{L}$ is the set of all points of a fine lattice Λ that are within the fundamental Voronoi region ν_1 of a coarse lattice Λ_1, i.e., $\mathcal{L} = \{\Lambda \cap \nu_1\}$.

In [18], it was shown that for every $P_1 \geq P_2 \geq 0$, there exist a sequence of n-dimensional lattice chains ${\Lambda_1}^n \subseteq {\Lambda_2}^n \subseteq {\Lambda_{c1}}^n$, as $n \to \infty$, $\sigma^2({\Lambda_1}^n) = P_1$ and $\sigma^2({\Lambda_2}^n) = P_2$. The rate of the nested lattice code $\mathcal{L}_2 = \{\Lambda^n \cap \nu_2\}$ associated with the lattice partition $\Lambda^n/{\Lambda_2}^n$, as $n \to \infty$ approaches

$$R_2 = \frac{1}{n} \log |\mathcal{L}_2| = \frac{1}{n} \log \frac{\mathrm{Vol}(\nu_2)}{\mathrm{Vol}(\nu)}, \tag{4.1}$$

while the coding rate of the nested lattice code $\mathcal{L}_1 = \{\Lambda^n \cap \nu_1\}$ associated with the lattice partition $\Lambda^n/{\Lambda_1}^n$, as $n \to \infty$ approaches

$$\begin{aligned} R_1 =& \frac{1}{n} \log |\mathcal{L}_1| = \frac{1}{n} \log \frac{\mathrm{Vol}(\nu_1)}{\mathrm{Vol}(\nu)} \\ =& \frac{1}{n} \log \frac{\mathrm{Vol}(\nu_1)}{\mathrm{Vol}(\nu_2)} + R_2 = R_2 + \frac{1}{2} \log \frac{P_1}{P_2}. \end{aligned} \tag{4.2}$$

We use an encoding/decoding scheme similar to the one used in [18].

State 9: Consider an n-dimensional lattice chain ${\Lambda_1}^n \subseteq {\Lambda_2}^n \subseteq {\Lambda_{c1}}^n$. Here, ${\Lambda_{c1}}^n$ is the coding lattice. Two codebooks $\mathcal{L}_1$, $\mathcal{L}_2$ defined as $\mathcal{L}_1 = \{\Lambda_{c1}^n \cap \nu_1\}$ and $\mathcal{L}_2 = \{\Lambda_{c1}^n \cap \nu_2\}$ are used for encoding, with the bigger codebook $\mathcal{L}_1$ being used for the node with the better link to R_1. Let $A \leftrightarrow R_1$ link be better than $B \leftrightarrow R_1$, i.e, $\gamma_{a1} \geq \gamma_{b1}$. Let the transmit power be P at all nodes. As $n \to \infty$, the second moments of Λ_1 and Λ_2 satisfy $\sigma^2({\Lambda_1}^n) \leftrightarrow \gamma_{a1}$ and $\sigma^2({\Lambda_2}^n) \leftrightarrow \gamma_{b1}$. Node A chooses a codeword $w_1 \in \mathcal{L}_1$ to transmit to R_1 while B chooses $w_2 \in \mathcal{L}_2$. The nodes add random dithers u_1 and u_2 to these codewords, where $u_1 \sim$ Unif(ν_1) and $u_2 \sim$ Unif(ν_2). The dither vectors are known at both the nodes and at the relays. Dithers are chosen to be independent of each other, and independent of the messages w_1,w_2. Hence, from the *crypto-lemma* of [18], each X_i will be independent of w_i and distributed as Unif(ν_i). The nodes A and B transmit

$$\begin{aligned} X_1 &= \frac{1}{h_{a1}}[(w_1 + u_1) \text{mod } \Lambda_1], \\ X_2 &= \frac{1}{h_{b1}}[(w_2 + u_2) \text{mod } \Lambda_2]. \end{aligned} \tag{4.3}$$

respectively. The transmit signals are pre-amplified to ensure the relay receives a noisy version of the sum $w_1 + w_2$.

The relay R_1 receives

$$Y_{R_1} = h_{a1}X_1 + h_{b1}X_2 + Z_{R_1}, \tag{4.4}$$

where Z_{R_1} is the additive gaussian noise. The relay computes αY_{R_1} and subtracts the dithers to obtain

$$\begin{aligned} \hat{Y}_{R_1} =& (\alpha Y_{R_1} - u_1 - u_2) \text{mod } \Lambda_1 \\ =& (\alpha h_{a1} X_1 + \alpha h_{b1} X_2 + \alpha Z_{R_1} - u_1 - u_2) \text{mod } \Lambda_1 \\ =& (h_{a1}X_1 - u_1 + h_{b1}X_2 - u_2 \\ & + (\alpha - 1)h_{a1}X_1 + (\alpha - 1)h_{b1}X_2 + \alpha Z_{R_1}) \text{mod } \Lambda_1 \\ =& (T_1 + \hat{Z}_{R_1}) \text{mod } \Lambda_1, \end{aligned}$$

where

$$
\begin{aligned}
T_1 =& (h_{a1}X_1 - u_1 + h_{b1}X_2 - u_2)\text{mod } \Lambda_1 \\
=& [(w_1 + u_1)\text{mod } \Lambda_1 + (w_2 + u_2)\text{mod} \\
& \Lambda_2 - u_1 - u_2]\text{mod } \Lambda_1 \\
=& (w_1 - Q_{\Lambda_1}(w_1 + u_1) + w_2 - Q_{\Lambda_2}(w_2 + u_2))\text{mod } \Lambda_1, \\
\hat{Z}_{R_1} =& -(1-\alpha)h_{a1}X_1 - (1-\alpha)h_{b1}X_2 + \alpha Z_{R_1}.
\end{aligned}
$$

It can be proved that the rate is uniquely maximised by choosing α to be the Minimum Mean Square Estimate (MMSE)coefficient,

$$
\alpha = \frac{\gamma_{a1} + \gamma_{b1}}{1 + \gamma_{a1} + \gamma_{b1}}.
$$

Here, $\hat{Z}_{R_1}$ is the effective noise at the relay, with variance

$$
\text{Var}(\hat{Z}_{R_1}) = \frac{(\gamma_{a1} + \gamma_{b1})N}{1 + \gamma_{a1} + \gamma_{b1}}. \tag{4.5}
$$

So, the effective noise variance has reduced, resulting in higher effective SNR at the relay.

According to *crypto-lemma*, T_1 is uniformly distributed over $\mathcal{L}_1$ and independent of $\hat{Z}_{R_1}$. Instead of decoding w_1 and w_2 individually, R_1 attempts to decode T_1 using *Euclidean Lattice Decoding*, by finding the codeword closest to $\hat{Y}_{R_1}$ in the coding lattice ${\Lambda_{c1}}^n$ using the quantizer $Q_{\lambda_{c1}}(\cdot)$.So, the relay decodes

$$
\hat{T}_1 = Q_{\lambda_{c1}}(\hat{Y}_{R_1}).
$$

An error occurs in the decoding of T_1 if $\hat{Z}_{R_1} \notin \nu_{c_1}$. So, probability of error can be written as Pr$(\hat{Z}_{R_1} \notin \nu_{c_1})$, which according to [16] vanishes as $n \to \infty$ if

$$
\sigma^2(\Lambda_{c1}) > \text{Var}(\hat{Z}_{R_1}). \tag{4.6}
$$

From (4.1), (4.2), (4.6), the rate constraints for State 1 can be written as

$$\begin{aligned} Z_{ar_1}^9 &\le \lambda_9 \left[\frac{1}{2}\log\left(\frac{\gamma_{a1}}{\gamma_{a1}+\gamma_{b1}}+\gamma_{a1}\right)\right]^+, \\ Z_{br_1}^9 &\le \lambda_9 \left[\frac{1}{2}\log\left(\frac{\gamma_{b1}}{\gamma_{a1}+\gamma_{b1}}+\gamma_{b1}\right)\right]^+. \end{aligned} \tag{4.7}$$

State 10: State 10 follows a similar lattice coding scheme as State 9, using a different lattice chain ${\Lambda_3}^n \subseteq {\Lambda_4}^n \subseteq {\Lambda_{c2}}^n$. We assume $A \leftrightarrow R_2$ link is better than $B \leftrightarrow R_2$ link. Nodes A and B choose messages w_3,w_4 from the codeword sets $\mathcal{L}_3 = \{\Lambda_{c2}^n \cap \nu_3\}$ and $\mathcal{L}_4 = \{\Lambda_{c2}^n \cap \nu_4\}$,and transmit them using dithers u_3 and u_4. The relay attempts to decode T_2 from the received vector Y_{R_2}, where

$$T_2 = (w_3 - Q_{\Lambda_3}(w_3+u_3) + w_4 - Q_{\Lambda_4}(w_4+u_4))\text{mod } \Lambda_3. \tag{4.8}$$

The relay computes

$$\hat{Y}_{R_2} = (\alpha Y_{R_2} - u_3 - u_4)\text{mod } \Lambda_3, \tag{4.9}$$

and obtains T_2 as

$$\hat{T}_2 = Q_{\lambda_{c2}}(\hat{Y}_{R_2}). \tag{4.10}$$

Rate constraints for state 10 can be written as

$$\begin{aligned} Z_{ar_2}^{10} &\le \lambda_{10} \left[\frac{1}{2}\log\left(\frac{\gamma_{a2}}{\gamma_{a2}+\gamma_{b2}}+\gamma_{a2}\right)\right]^+, \\ Z_{br_2}^{10} &\le \lambda_{10} \left[\frac{1}{2}\log\left(\frac{\gamma_{b2}}{\gamma_{a2}+\gamma_{b2}}+\gamma_{b2}\right)\right]^+. \end{aligned} \tag{4.11}$$

State 13: In state 13, both the relays simultaneously transmit to both nodes A and B. Relay R_1 generates a codebook C_{R_1} consisting of $|\mathcal{L}_1|$ n-length sequences, with each element being i.i.d having distribution $\mathcal{N}(0, P)$. We assume that the relays make no error in decoding T_1 and T_2 in the first two states. Since T_1 is uniformly distributed over $\mathcal{L}_1$, for every $T_1 = t_1 \in \mathcal{L}_1$, the relay chooses to transmit a particular $X_{R_1}(t_1) \in C_{R_1}$. Similarly, R_2 generates a random codebook C_{R_2} consisting of $|\mathcal{L}_3|$ n-length sequences,with each element being i.i.d having distribution $\mathcal{N}(0, P)$ and for every $T_2 = t_2 \in \mathcal{L}_3$, the relay broadcasts a particular $X_{R_2}(t_2) \in C_{R_2}$. This results in Multiple Access Channels at nodes A and B. Node A receives

$$Y_a = h_{a1}X_{R_1} + h_{a2}X_{R_2} + Z_a, \tag{4.12}$$

from which it decodes X_{R_1} and X_{R_2} separately. Since there is a one-to-one correspondence between the elements of $\mathcal{L}_1$ and C_{R_1}, A can obtain $\hat{T}_1$ from X_{R_1}. Also, A can obtain $\hat{T}_2$ from X_{R_2} due to the one-to-one correspondence between the elements of $\mathcal{L}_3$ and C_{R_2}. Similarly, B can obtain $\tilde{T}_1$ and $\tilde{T}_2$ from the received vector

$$Y_b = h_{b1}X_{R_1} + h_{b2}X_{R_2} + Z_b. \tag{4.13}$$

w_1 and w_3 are messages transmitted by A to the relays, and hence A has apriori knowledge of w_1 and w_3. The dither vectors are also known at all the nodes. This apriori knowledge can be used as side-information for decoding [20]. Using the knowledge of w_1, A can decode w_2 from $\hat{T}_1$ as

$$w_2 = [\hat{T}_1 - w_1]\text{mod } \Lambda_2. \tag{4.14}$$

Using w_3, A can decode w_4 from $\hat{T}_2$ as

$$w_4 = [\hat{T}_2 - w_3]\text{mod } \Lambda_4. \tag{4.15}$$

Similarly, using its apriori knowledge of w_2, w_4 and the dithers, node B can decode w_1 and w_3 from T_1 and T_2 as

$$\begin{aligned} w_1 &= [\tilde{T}_1 - w_2 + Q_{\Lambda_2}(w_2 + u_2)]\text{mod } \Lambda_1, \\ w_3 &= [\tilde{T}_2 - w_4 + Q_{\Lambda_4}(w_4 + u_4)]\text{mod } \Lambda_3. \end{aligned} \tag{4.16}$$

From (4.7), (4.11) the rate constraints for the CF-CMAC protocol can be summarised as follows:

State 9:

$$\begin{aligned} Z^9_{ar_1} &\leq \lambda_9 \left[C(\gamma_{a1} - \frac{\gamma_{b1}}{\gamma_{a1} + \gamma_{b1}}) \right]^+, \\ Z^9_{br_1} &\leq \lambda_9 \left[C(\gamma_{b1} - \frac{\gamma_{a1}}{\gamma_{a1} + \gamma_{b1}}) \right]^+. \end{aligned} \tag{4.17}$$

State 10:

$$
\begin{aligned}
Z^{10}_{ar_2} &\leq \lambda_{10}\left[C(\gamma_{a2} - \frac{\gamma_{b2}}{\gamma_{a2}+\gamma_{b2}})\right]^+, \\
Z^{10}_{br_2} &\leq \lambda_{10}\left[C(\gamma_{b2} - \frac{\gamma_{a2}}{\gamma_{a2}+\gamma_{b2}})\right]^+.
\end{aligned} \tag{4.18}
$$

State 13: From [20], we can write the rate constraints for the broadcasting relays with apriori knowledge of some messages as

$$
\begin{aligned}
Z^{13}_{r_1a} &\leq \lambda_{13}C(\gamma_{a1}), \\
Z^{13}_{r_1b} &\leq \lambda_{13}C(\gamma_{b1}).
\end{aligned} \tag{4.19}
$$

and

$$
\begin{aligned}
Z^{13}_{r_2a} &\leq \lambda_{13}C(\gamma_{a2}), \\
Z^{13}_{r_2b} &\leq \lambda_{13}C(\gamma_{b2}).
\end{aligned} \tag{4.20}
$$

at R_1 and R_2. The MACs at A and B impose two additional constraints on the sum rates as follows:

$$
\begin{aligned}
Z^{13}_{r_1a} + Z^{13}_{r_2a} &\leq \lambda_{13}C(\gamma_{a1}+\gamma_{a2}), \\
Z^{13}_{r_1b} + Z^{13}_{r_2b} &\leq \lambda_{13}C(\gamma_{b1}+\gamma_{b2}).
\end{aligned} \tag{4.21}
$$

Equating the information received at a relay from one node to the information forwarded by it to the other node, gives us four equality constraints.

$$
\begin{aligned}
Z^{9}_{ar_1} &= Z^{13}_{r_1b}, \\
Z^{9}_{br_1} &= Z^{13}_{r_1a}, \\
Z^{10}_{ar_2} &= Z^{13}_{r_2b}, \\
Z^{10}_{br_2} &= Z^{13}_{r_2a}.
\end{aligned} \tag{4.22}
$$

The rates of information transfer between the end nodes A and B in the two directions are

$$
\begin{aligned}
R_a &= Z^{9}_{ar_1} + Z^{10}_{ar_2}, \\
R_b &= Z^{9}_{br_1} + Z^{10}_{br_2}.
\end{aligned} \tag{4.23}
$$

The achievable rate region can be obtained by taking $R_b = kR_a$ and solving the linear program $\max_{\{\lambda_i\}} R_b$ with (4.17) to (4.22) and $\sum_{i=9,10,13} \lambda_i = 1$ as constraints, for various values of k.

Time-sharing between CF-CMAC and Two-way MDF: The CF-CMAC protocol achieves some rate-pairs that the two-way MDF protocol cannot achieve. Time-sharing between CF-CMAC and two-way MDF can be used to achieve all convex combinations of rate-pairs achieved by the two protocols. Such a protocol would used $8 + 3 = 11$ states.

4.3 CF-BC protocol

The CF-BC (Compute and Forward-Broadcast Channel) protocol uses States 9-12. Only one relay is used in each state. States 9 and 10 are used the same way as in the CF-CMAC protocol, i.e, A and B simultaneously transmit to R_1 in State 9 and to R_2 in State 10. The relays R_1 and R_2 compute the sum of messages received by them in states 9 and 10 and forward them to the end nodes in Broadcast States 11 and 12 respectively. The nodes can decode the messages meant from them using their apriori knowledge of the messages sent by them to the relays. This scheme is basically a time sharing of two-way relaying with one relay. Rate constraints for State 9 and State 10 are same as those in CF-CMAC protocol. The rate constraints for states 11 and 12 are:

$$\begin{aligned} F^{11}_{r_1a} &\leq \lambda_{11} C(\gamma_{a1}), \\ F^{11}_{r_1b} &\leq \lambda_{11} C(\gamma_{b1}), \end{aligned} \tag{4.24}$$

$$\begin{aligned} F^{12}_{r_2a} &\leq \lambda_{12} C(\gamma_{a2}), \\ F^{12}_{r_2b} &\leq \lambda_{12} C(\gamma_{b2}). \end{aligned} \tag{4.25}$$

14 state protocol

A decode and forward protocol using all the 14 possible states of the diamond relay channel was also considered. In this, the messages transmitted in each state are decoded at the destination at the end of the state. It was found that using 14 states does not provide any significant increase in rate region as compared to the Two-way MDF protocol that uses only 8 states. This could be because of decode and forward not being the best relaying scheme for states 13 and 14, which are 2x2 interference networks. Compute and forward seems to be the best relaying scheme to achieve rates close capacity.

The achievable rate regions of all the three protocols are analysed and compared in Chapter 7.

CHAPTER 5

OUTER BOUND FOR THE CAPACITY REGION OF THE GAUSSIAN DIAMOND RELAY CHANNEL

In this chapter, we derive outer bounds for the capacity region of the Gaussian Diamond Relay Channel without direct links. The bound is derived using the half-duplex cut-set bound for the capacity of a single flow in an arbitrary half-duplex relay network in [21]. This outer bound is valid for all relaying schemes, because the derivation does not involve any assumptions about the type of relaying used.

From [21], for any general network with M states and a fraction of time μ_i in state i, any achievable rate R of information flow is bounded as

$$R \leq \min_{S} \sum_{i=1}^{M} \mu_i I(X^S; Y^{S^c} | X^{S^c}, i), \tag{5.1}$$

where R is the rate from source to destination node with the source in a subset of nodes S and the destination in S^c. The set S defines a cut that separates source and destination.

There are two flows in the two-way diamond channel, R_a from A to B and R_b form B to A. The cutset bound is applied to the two flows cuts $\{a\}$,$\{a, r_1\}$,$\{a, r_2\}$,$\{a, r_1, r_2\}$ for bounding R_a and the cuts $\{b\}$,$\{b, r_1\}$,$\{b, r_2\}$,$\{b, r_1, r_2\}$ for bounding R_b.

We need not consider all 14 states of Figure 3.2 for deriving the outer bound. 8 of the states (3,4 and 7-12) that use only three of the four available nodes are actually a part of either State 13 or State 14. So, we need to consider only 6 states for writing the cut-set bound: 14, 13, 1, 2, 5, 6, with the fraction of time the network is in these states being denoted as $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6$, respectively.

Using the eight cuts mentioned above cuts, we obtain the following inequations:

$$R_a \leq \min \{R_{a1}, R_{a2}, R_{a3}, R_{a4}\},$$

where

$$
\begin{aligned}
R_{a1} =& \mu_1 I(X_a; Y_1, Y_2|X_b) + \mu_3 I(X_a; Y_1|X_2) \\
& + \mu_4 I(X_a; Y_2|X_1), \\
R_{a2} =& \mu_1 I(X_a; Y_2|X_b) + \mu_2 I(X_1; Y_b|X_2) \\
& + \mu_4 (I(X_a; Y_2) + I(X_1; Y_b)), \\
R_{a3} =& \mu_1 I(X_a; Y_1|X_b) + \mu_2 I(X_2; Y_b|X_1) \\
& + \mu_3 (I(X_a; Y_1) + I(X_2; Y_b)), \\
R_{a4} =& \mu_2 I(X_1, X_2; Y_b) + \mu_3 I(X_2; Y_b) \\
& + \mu_4 I(X_1; Y_b),
\end{aligned}
$$

and

$$
R_b \leq \min\{R_{b1}, R_{b2}, R_{b3}, R_{b4}\},
$$

where

$$
\begin{aligned}
R_{b1} =& \mu_1 I(X_b; Y_1, Y_2|X_a) + \mu_5 I(X_b; Y_1|X_2) \\
& + \mu_6 I(X_b; Y_2|X_1), \\
R_{b2} =& \mu_1 I(X_b; Y_2|X_a) + \mu_2 I(X_1; Y_a|X_2) \\
& + \mu_6 (I(X_b; Y_2) + I(X_1; Y_a)), \\
R_{b3} =& \mu_1 I(X_b; Y_1|X_a) + \mu_2 I(X_2; Y_a|X_1) \\
& + \mu_5 (I(X_b; Y_1) + I(X_2; Y_b)), \\
R_{b4} =& \mu_2 I(X_1, X_2; Y_a) + \mu_5 I(X_2; Y_b a) \\
& + \mu_6 I(X_1; Y_a).
\end{aligned}
$$

The mutual information terms in the above equations can be further bounded as in [17]. For example, $I(X_b; Y_1, Y_2|X_a) \leq C(\gamma_{b1}+\gamma_{b2})$ and $I(X_1, X_2; Y_b) \leq C((\sqrt{\gamma_{b1}}+\sqrt{\gamma_{b2}})^2)$.

This results in the following outer bound:

$$
\begin{aligned}
&R_a \leq \mu_1 C(\gamma_{a1} + \gamma_{a2}) + \mu_3 C(\gamma_{a1}) + \mu_4 C(\gamma_{a2}), \\
&R_a \leq \mu_1 C(\gamma_{a2}) + \mu_2 C(\gamma_{b1}) + \mu_4 (C(\gamma_{a2}) + C(\gamma_{b1})), \\
&R_a \leq \mu_1 C(\gamma_{a1}) + \mu_2 C(\gamma_{b2}) + \mu_3 (C(\gamma_{a1}) + C(\gamma_{b2})), \\
&R_a \leq \mu_2 C((\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}})^2) + \mu_3 C(\gamma_{b2}) + \mu_4 C(\gamma_{b1}), \\
&R_b \leq \mu_1 C(\gamma_{b1} + \gamma_{b2}) + \mu_5 C(\gamma_{b1}) + \mu_6 C(\gamma_{b2}), \\
&R_b \leq \mu_1 C(\gamma_{b2}) + \mu_2 C(\gamma_{a1}) + \mu_6 (C(\gamma_{a1}) + C(\gamma_{b2})), \\
&R_b \leq \mu_1 C(\gamma_{b1}) + \mu_2 C(\gamma_{a2}) + \mu_5 (C(\gamma_{b1}) + C(\gamma_{a2})), \\
&R_b \leq \mu_2 C((\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}})^2) + \mu_5 C(\gamma_{a2}) + \mu_6 C(\gamma_{a1}), \\
&\sum_{i=1}^{6} \mu_i = 1, \mu_i \geq 0.
\end{aligned}
\tag{5.2}
$$

The boundary of the above capacity region can be computed by solving the following linear program for each $k \geq 0$:

$$\max_{R_a, \{\mu_i\}} R_a,$$

subject to $R_a = kR_b$ and all the constraints in (5.2).

CHAPTER 6

THE GAUSSIAN TWO-WAY DIAMOND CHANNEL WITH DIRECT LINKS

6.1 The Gaussian Diamond channel with direct link between nodes

The diamond relay channel with direct link between the end nodes is shown in Figure 6.1. The system model is similar to Section 3.1, but for an additional link between A and B. The SNR of the direct link is denoted as $\gamma_{ab} = \frac{h^2{}_{ab}P}{N}$. This system has not been considered in literature.

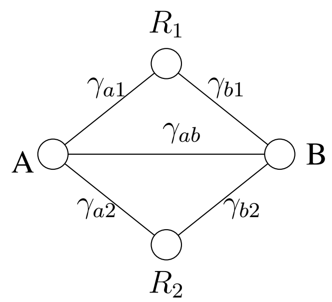

Figure 6.1: Gaussian Diamond Channel with A-B Direct Link

6.1.1 Achievable rate region : The two-relay Co-MABC protocol

The system in Fig.6.1 can be seen as a simple extension of the two-way relay channel with direct link, by adding another relay. The CF-CMAC and CF-BC protocols can be used for this model also, but they do not use the direct link. The direct link can be exploited to increase the time-efficiency and throughput of this system. The Cooperative MABC (CoMABC) protocol proposed in [3] for the three-node two way relay case that makes use of the direct link to increase the achievable rate region. This is a three state protocol, in which the first state is a MAC, both end nodes A and B transmit to the relay. The second state is a BC from the relay. If the link A to relay is better than the one from B, then A may transmit more bits in the first state and B may receive at much lower rate in the second state. This is compensated using a third co-operative state in after A finishes decoding in the broadcast state. In the third state, A and the relay together transmit to B. A may retransmit some information to help B in decoding, or may choose to transmit altogether new information. The same scheme can be applied twice, once at each relay in case of the gaussian diamond channel with direct link. We call this the The Two-relay Co-MABC protocol.

Assuming that the links from A to the relays are better than those from B to the relays, the achievable rate region is the closure of the set of all points (R_a, R_b) satisfying following constraints:

$$\begin{aligned} R_a =& R_{a1} + R_{a2},\ R_b = R_{a1} + R_{a2} \text{ where} \\ & R_{a1} \leq \min\{\lambda_1 R^*_{ar1} + \lambda_3 \mathcal{C}(\gamma_{ab}), \lambda_2 \mathcal{C}(\gamma_{b1}) + \lambda_3 \mathcal{C}(\gamma_{b1} + \gamma_{ab})\}, \\ & R_{b1} \leq \min\{\lambda_1 R^*_{br1}, \lambda_2 \mathcal{C}(\gamma_{a1})\}, \\ & R_{a2} \leq \min\{\lambda_4 R^*_{ar2} + \lambda_6 \mathcal{C}(\gamma_{ab}), \lambda_5 \mathcal{C}(\gamma_{b2}) + \lambda_6 \mathcal{C}(\gamma_{b2} + \gamma_{ab})\}, \\ & R_{b2} \leq \min\{\lambda_4 R^*_{br2}, \lambda_5 \mathcal{C}(\gamma_{a2})\}, \\ R^*_{ar1} =& \left[C(\gamma_{a1} - \frac{\gamma_{b1}}{\gamma_{a1} + \gamma_{b1}}) \right]^+, R^*_{br1} = \left[C(\gamma_{b1} - \frac{\gamma_{b1}}{\gamma_{a1} + \gamma_{b1}}) \right]^+, \\ R^*_{ar2} =& \left[C(\gamma_{a2} - \frac{\gamma_{b2}}{\gamma_{a2} + \gamma_{b2}}) \right]^+, R^*_{br2} = \left[C(\gamma_{b2} - \frac{\gamma_{b2}}{\gamma_{a2} + \gamma_{b2}}) \right]^+. \end{aligned} \tag{6.1}$$

where $[x]^+ \triangleq max(x, 0)$.

6.1.2 Outer bound for capacity region

The states to be considered for writing the outer bound are shown in Figure 6.2. Proceeding in the same way as in Chapter 5, the outer bound to the capacity region can be computed by solving for each $k \geq 0$:

$$\max_{R_a, \{\mu_i\}} R_a,$$

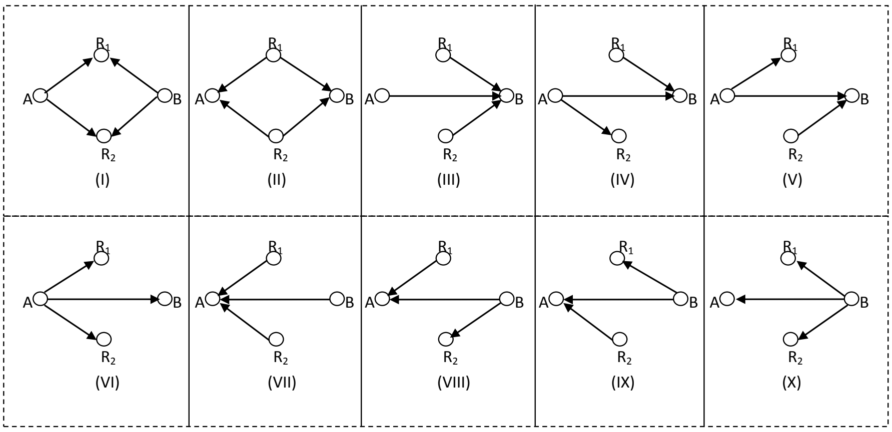

Figure 6.2: States in a Gaussian Diamond Relay network with direct link between nodes A and B.

subject to $R_a = kR_b$ and the constraints

$$
\begin{aligned}
R_a \leq& \lambda_1 C(\gamma_{a1} + \gamma_{a2}) + \lambda_3 C(\gamma_{ab}) + \lambda_4 C(\gamma_{a2} + \gamma_{ab}) \\
&+ \lambda_5 C(\gamma_{a1} + \gamma_{ab}) + \lambda_6 C(\gamma_{a1} + \gamma_{a2} + \gamma_{ab}), \\
R_a \leq& \lambda_1 C(\gamma_{a2}) + \lambda_2 C(\gamma_{b1}) + \lambda_3 C((\sqrt{\gamma_{b1}} + \sqrt{\gamma_{ab}})^2) \\
&+ \lambda_4 (C(\gamma_{a2}) + C(\gamma_{a2} + \gamma_{ab})) + \lambda_5 C(\gamma_{ab}) + \lambda_6 C(\gamma_{a2} + \gamma_{ab}), \\
R_a \leq& \lambda_1 C(\gamma_{a1}) + \lambda_2 C(\gamma_{b2}) + \lambda_3 C((\sqrt{\gamma_{b2}} + \sqrt{\gamma_{ab}})^2) \\
&+ \lambda_4 C(\gamma_{ab}) + \lambda_5 (C(\gamma_{a1}) + C(\gamma_{b2} + \gamma_{ab})) + \lambda_6 C(\gamma_{a1} + \gamma_{ab}), \\
R_a \leq& \lambda_2 C((\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}})^2) + \lambda_3 C((\sqrt{\gamma_{b1}} + \sqrt{\gamma_{b2}} + \sqrt{\gamma_{ab}})^2) \\
&+ \lambda_4 C((\sqrt{\gamma_{b1}} + \sqrt{\gamma_{ab}})^2) + \lambda_5 C((\sqrt{\gamma_{b2}} + \sqrt{\gamma_{ab}})^2) + \lambda_6 C(\gamma_{ab}),
\end{aligned}
$$

$$
\begin{aligned}
R_b \leq& \lambda_1 C(\gamma_{b1} + \gamma_{b2}) + \lambda_7 C(\gamma_{ab}) + \lambda_8 C(\gamma_{b2} + \gamma_{ab}) \\
&+ \lambda_9 C(\gamma_{b1} + \gamma_{ab}) + \lambda_{10} C(\gamma_{b1} + \gamma_{b2} + \gamma_{ab}), \\
R_b \leq& \lambda_1 C(\gamma_{b2}) + \lambda_2 C(\gamma_{a1}) + \lambda_7 C((\sqrt{\gamma_{a1}} + \sqrt{\gamma_{ab}})^2) \\
&+ \lambda_8 (C(\gamma_{b2}) + C(\gamma_{a1} + \gamma_{ab})) + \lambda_9 C(\gamma_{ab}) + \lambda_{10} C(\gamma_{b2} + \gamma_{ab}), \\
R_b \leq& \lambda_1 C(\gamma_{b1}) + \lambda_2 C(\gamma_{a2}) + \lambda_7 C((\sqrt{\gamma_{a2}} + \sqrt{\gamma_{ab}})^2) \\
&+ \lambda_8 C(\gamma_{ab}) + \lambda_9 (C(\gamma_{b1}) + C(\gamma_{a2} + \gamma_{ab})) + \lambda_{10} C(\gamma_{b1} + \gamma_{ab}), \\
R_b \leq& \lambda_2 C((\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}})^2) + \lambda_7 C((\sqrt{\gamma_{a1}} + \sqrt{\gamma_{a2}} + \sqrt{\gamma_{ab}})^2) \\
&+ \lambda_8 C((\sqrt{\gamma_{a1}} + \sqrt{\gamma_{ab}})^2) + \lambda_9 C((\sqrt{\gamma_{a2}} + \sqrt{\gamma_{ab}})^2) + \lambda_{10} C(\gamma_{ab}), \\
&\sum_{i=1}^{10} \lambda_i = 1, \lambda_i \geq 0.
\end{aligned}
$$

6.2 The Gaussian Diamond channel with direct link between relays

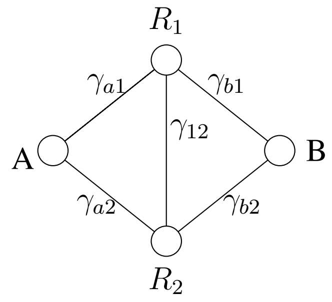

Figure 6.3: Gaussian Diamond Channel with R_1-R_2 Direct Link.

Figure 6.3 shows the Gaussian Diamond channel with direct link between the relays. The two relays are no longer non-interfering. This scenario has been considered in [14] as the alternating two path relay channel and a Decode and Forward strategy has been proposed for the same. When R_1 is in transmit mode and R_2 is in receive mode, R_2 can treat the information from R_1 in two ways. One way is to consider it as interference, and other is to consider it as a message to be decoded and forwarded to the destination in the next state. The strategy proposed in [14] is a combination of these two schemes.

6.2.1 Achievable rate region using Alternating two path relay channels

An achievable rate region for this channel can be obtained using States 4, 6, 8 and 10 of Figure 6.4. Consider the flow from A to B. In State 4, A transmits to R_2 while R_1 transmits to B. R_2 decodes the message from A, and in addition decodes the message from R_1 too, considering it as data to be forwarded to B in the next state. So R_2 acts as a relay for both A and R_1. In state 6, A transmits new information to R_1, R_2 transmits to B. The message transmitted by R_2 will be a combination of the messages received from A and R_2 and the message to be sent to R_1. States 8 and 10 act in the same way for the flow in the opposite direction, R_b. The encoding and decoding schemes used are Block Markov encoding and Sliding window decoding.

The achievable rate region is the closure of the set of all points (R_a, R_b) satisfying following constraints[14]:

$$
\begin{aligned}
R_a = & R_{a1} + R_{a2}, R_b = R_{a1} + R_{a2} \text{ where} \\
& R_{a1} \leq \lambda_1 \mathcal{C}(\alpha_1 \gamma_{a2}), \\
& R_{a1} \leq \lambda_1 \mathcal{C}\left(\frac{\beta_1 \gamma_{b1}}{1+(1-\beta_1)\gamma_{b1}}\right) + \lambda_2 \mathcal{C}((1-\beta_2)\gamma_{b2}), \\
& R_{a2} \leq \lambda_2 \mathcal{C}(\alpha_2 \gamma_{a2}), \\
& R_{a2} \leq \lambda_2 \mathcal{C}\left(\frac{\beta_2 \gamma_{b2}}{1+(1-\beta_2)\gamma_{b2}}\right) + \lambda_1 \mathcal{C}((1-\beta_1)\gamma_{b1}), \\
& R_{b1} \leq \lambda_3 \mathcal{C}(\alpha_3 \gamma_{b2}), \\
& R_{b1} \leq \lambda_3 \mathcal{C}\left(\frac{\beta_3 \gamma_{a1}}{1+(1-\beta_3)\gamma_{a1}}\right) + \lambda_4 \mathcal{C}((1-\beta_4)\gamma_{a2}), \\
& R_{b2} \leq \lambda_4 \mathcal{C}(\alpha_4 \gamma_{b2}), \\
& R_{b2} \leq \lambda_4 \mathcal{C}\left(\frac{\beta_4 \gamma_{a2}}{1+(1-\beta_4)\gamma_{a2}}\right) + \lambda_3 \mathcal{C}((1-\beta_3)\gamma_{a1}).
\end{aligned}
\tag{6.2}
$$

6.2.2 Outer bound for capacity region

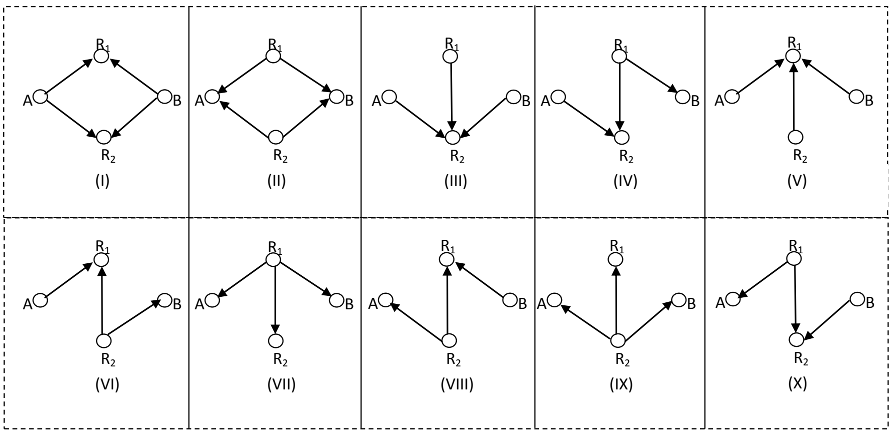

Figure 6.4: States in a Gaussian Diamond Relay network with direct link between relays.

The 10 states of this network to be considered for obtaining the capacity upper bound are shown in Figure 6.4. The outer bound to the capacity region can be computed by solving:

$$\max_{R_a, \{\mu_i\}} R_a,$$

subject to $R_a = kR_b$ for each $k \geq 0$ and the following constraints:

$$
\begin{aligned}
R_a \leq & \lambda_1 C(\gamma_{a1}+\gamma_{a2}) + \lambda_3 C(\gamma_{a2}) + \lambda_4 C(\gamma_{a2}) + \lambda_5 C(\gamma_{a1}) + \lambda_6 C(\gamma_{a1}), \\
R_a \leq & \lambda_1 C(\gamma_{a2}) + \lambda_2 C(\gamma_{b1}) + \lambda_3 C((\sqrt{\gamma_{a2}}+\sqrt{\gamma_{12}})^2) + \lambda_4 (C(\gamma_{a2}) + C(\gamma_{b1}+\gamma_{12})) \\
& + \lambda_7 C(\gamma_{b1}+\gamma_{12}) + \lambda_{10} C(\gamma_{12}), \\
R_a \leq & \lambda_1 C(\gamma_{a1}) + \lambda_2 C(\gamma_{b2}) + \lambda_5 C((\sqrt{\gamma_{a1}}+\sqrt{\gamma_{12}})^2) + \lambda_6 (C(\gamma_{a1}) + C(\gamma_{b2}+\gamma_{12})) \\
& + \lambda_9 C(\gamma_{b2}+\gamma_{12}) + \lambda_8 C(\gamma_{12}), \\
R_a \leq & \lambda_2 C((\sqrt{\gamma_{b1}}+\sqrt{\gamma_{b2}})^2) + \lambda_4 C(\gamma_{b1}) + \lambda_6 C(\gamma_{b2}) + \lambda_7 C(\gamma_{b1}) + \lambda_9 C(\gamma_{b2}), \\
R_b \leq & \lambda_1 C(\gamma_{b1}+\gamma_{b2}) + \lambda_3 C(\gamma_{b2}) + \lambda_5 C(\gamma_{b1}) + \lambda_8 C(\gamma_{b1}) + \lambda_{10} C(\gamma_{b2}), \\
R_b \leq & \lambda_1 C(\gamma_{b2}) + \lambda_2 C(\gamma_{a1}) + \lambda_3 C((\sqrt{\gamma_{b2}}+\sqrt{\gamma_{12}})^2) + \lambda_4 (C(\gamma_{12}) + \lambda_7 C(\gamma_{a1}+\gamma_{12}) \\
& + \lambda_{10} (C(\gamma_{b2}) + C(\gamma_{a1}+\gamma_{12})), \\
R_b \leq & \lambda_1 C(\gamma_{b1}) + \lambda_2 C(\gamma_{a2}) + \lambda_5 C((\sqrt{\gamma_{b1}}+\sqrt{\gamma_{12}})^2) + \lambda_6 (C(\gamma_{12}) + \lambda_8 (C(\gamma_{b1}) + C(\gamma_{a2}+\gamma_{12})) \\
& + \lambda_9 C(\gamma_{a2}+\gamma_{12}), \\
R_b \leq & \lambda_2 C((\sqrt{\gamma_{a1}}+\sqrt{\gamma_{a2}})^2) + \lambda_7 C(\gamma_{a1}) + \lambda_8 C(\gamma_{a2}) + \lambda_9 C(\gamma_{a2}) + \lambda_{10} C(\gamma_{a1}).,\\
& \sum_{i=1}^{10} \lambda_i = 1, \lambda_i \geq 0.
\end{aligned}
$$

CHAPTER 7

NUMERICAL RESULTS

In this section, we compare the analytical outer bound derived in Chapter 2 with the outer bound of [10]. We also compare the achievable rate regions of the various protocols proposed with the corresponding outer bounds.

7.1 Comparison of analytical and numerical outer bounds

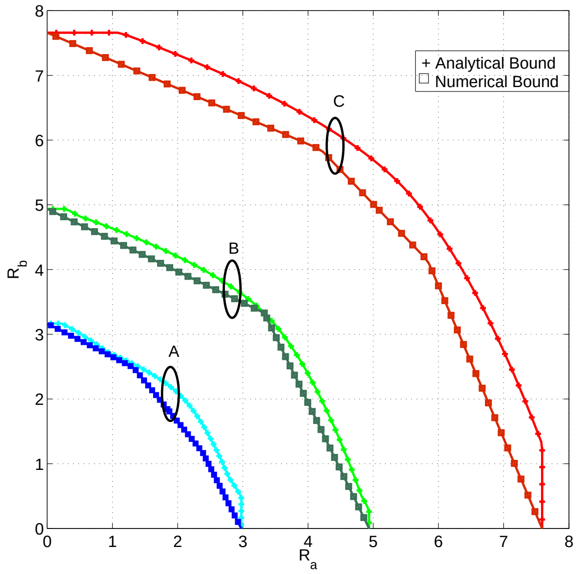

Figure 7.1: Comparison of analytical and numerical outer bounds: A:$\gamma_1 = 10d$B,$\gamma_2 =$ 15dB,$\gamma_3 =$ 3dB, B:$\gamma_1 = 20d$B,$\gamma_2 =$ 20dB,$\gamma_3 =$ 8dB, and C:$\gamma_1 =$ $30d$B,$\gamma_2 =$ 35dB,$\gamma_3 =$ 13dB.

Figure 7.1 shows a comparison of the numerical and analytical outer bounds for three channel conditions A: $\gamma_1 = 10d$B,$\gamma_2 =$ 15dB,$\gamma_3 =$ 3dB, B: $\gamma_1 = 20d$B,$\gamma_2 =$ 20dB,$\gamma_3 =$ 8dB, and C: $\gamma_1 = 30d$B,$\gamma_2 =$ 35dB,$\gamma_3 =$ 13dB. The numerical outer bound is obtained by solving the linear program in (2.1). The analytical bound is found to be close to the numerical bound in all cases. In case B, where $\gamma_1 = \gamma_2$, the bounds match for $k = 1$ as expected.

7.2 Comparison of rate regions

7.2.1 Without direct link

Comparison of achievable rate regions of the different protocols with outer bound is shown for three different channel conditions: (1) $\gamma_{a1} = 15$ dB, $\gamma_{b1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b2} = 15$ dB, (2) $\gamma_{a1} = 10$ dB, $\gamma_{b1} = 12$ dB, $\gamma_{a2} = 14$ dB, $\gamma_{b2} = 16$ dB, and (3) $\gamma_{a1} = 30$ dB, $\gamma_{b1} = 20$ dB, $\gamma_{a2} = 3$ dB, $\gamma_{b2} = 4$ dB.

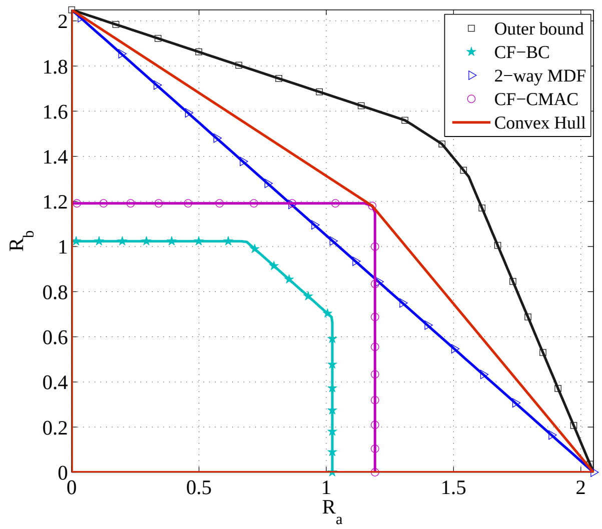

Figure 7.2: Comparison of rate regions: $\gamma_{a1} = 15$ dB, $\gamma_{b1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b2} = 15$ dB.

Figure 7.2 shows the comparison of rate regions for channel condition 1. Here, $C(\gamma_{a1})C(\gamma_{b2}) = C(\gamma_{b1})C(\gamma_{a2})$. According to [13], in this channel condition the one-way MDF protocol achieves capacity. This can be seen from the two-way MDF region meeting the outer bound on the axes($R_a = 0$ and $R_b = 0$) where it corresponds to one-way MDF. But away from the two axes, there is a significant gap from the outer bound. The CF-BC protocol, using only one relay at a time has a smaller rate region than two-way MDF. However, the CF-CMAC protocol is able to achieve rate-pairs outside the rate region of the two-way MDF protocol. The convex combination of the CF-CMAC and two-way MDF protocol rate regions is also shown (labelled "convex hull"). Any point in this convex hull can be achieved by time-sharing the CF-CMAC and two-way MDF protocols.

Figure 7.3 shows the comparison of rate regions for asymmetric channel condition

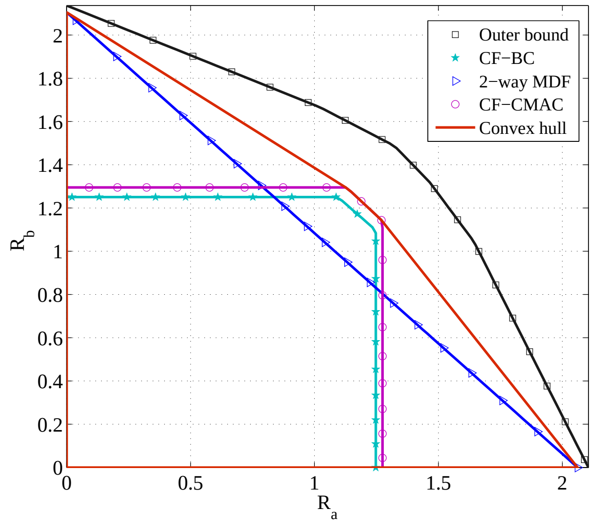

Figure 7.3: Comparison of rate regions: $\gamma_{a1} = 10$ dB, $\gamma_{b1} = 12$ dB, $\gamma_{a2} = 14$ dB, $\gamma_{b2} = 16$ dB.

2. In this scenario, both the CF-BC and CF-CMAC protocols achieve rate-pairs outside the two-way MDF rate region.

Figure 7.4 shows the comparison of rate regions for channel condition 3. In this scenario, the links to relay R_1 are significantly better than the links to R_2 in terms of SNR. Therefore, this scenario is closer to a one-relay system. Both the CF-BC and CF-CMAC protocols achieve rate-pairs outside the two-way MDF rate region and the convex-hull is significantly close to the outer bound.

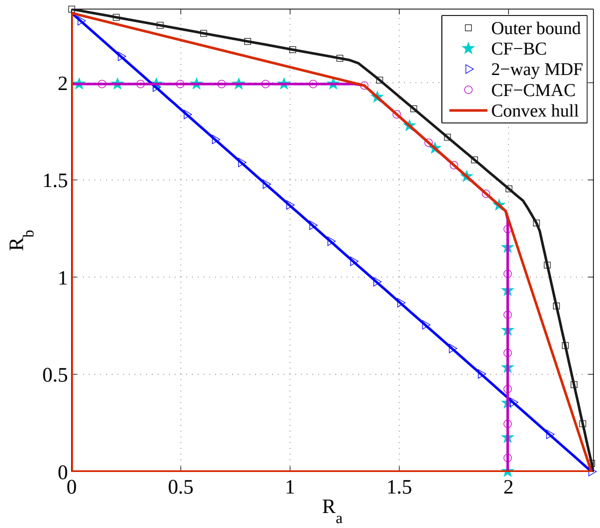

Figure 7.4: Comparison of rate regions: $\gamma_{a1} = 30$ dB, $\gamma_{b1} = 20$ dB, $\gamma_{a2} = 3$ dB, $\gamma_{b2} = 4$ dB.

In general, the one-way MDF protocol is close to capacity for the one-way diamond channel. So by time-sharing, one can obtain rates close to capacity near the axes. However, when the desired two-way rates are nearly equal, it is evident that time-sharing of one-way protocols is far from optimal. In such a case, mixing of the two flows is better, as is evident from the proposed CF-CMAC and CF-BC protocols achieving much better rates along the $R_a = R_b$ line.

7.2.2 With direct link between A and B

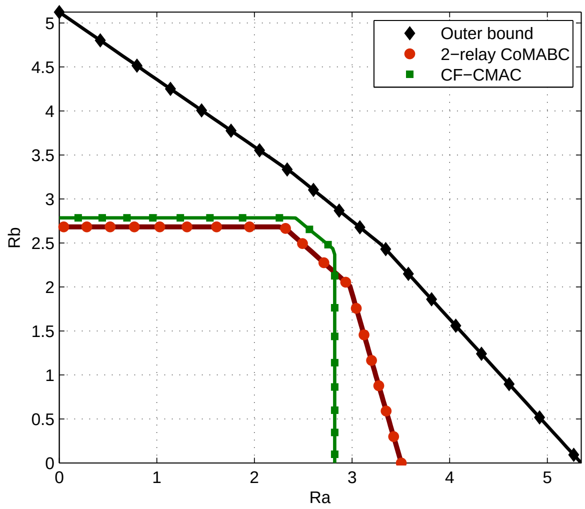

Figure 7.5: Comparison of rate regions: $\gamma_{a1} = 35$ dB, $\gamma_{b1} = 30$ dB, $\gamma_{a2} = 35$ dB, $\gamma_{b2} = 30$ dB, $\gamma_{ab} = 15$ dB.

Figure 7.5 shows the comparison of rate regions achieved using the two-relay CoMABC protocol and CF-CMAC with the outer bound for the case $\gamma_{a1} = 35$ dB, $\gamma_{b1} = 30$ dB, $\gamma_{a2} = 35$ dB, $\gamma_{b2} = 30$ dB, $\gamma_{ab} = 15$ dB. The CoMABC protocol is proposed to make best use of asymmetric channel conditions, hence it is observed to have a larger rate region than CF-CMAC. Also, CoMABC is optimized to maximize sum rate. Therefore, it performs well near the maximum sum rate points, while it is away from the outer bound near the axes. This protocol uses only 4 of the 14 possible states of the diamond channel. The other states can be exploited to increase the rate region near the axes.

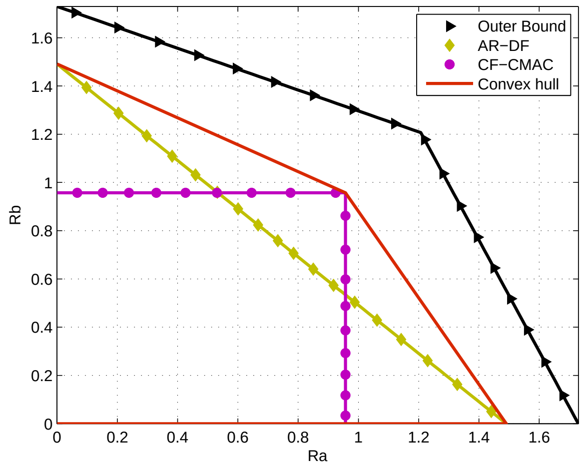

Figure 7.6: Comparison of rate regions: $\gamma_{a1} = 10$ dB, $\gamma_{b1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b2} = 10$ dB, $\gamma_{12} = 15$ dB.

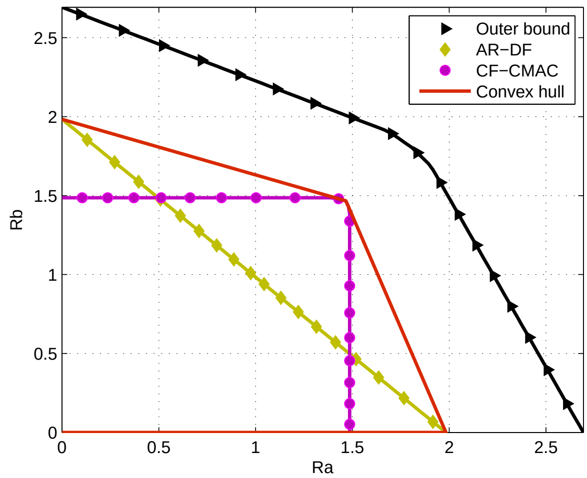

Figure 7.7: Comparison of rate regions: $\gamma_{a1} = 20$ dB, $\gamma_{b1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b2} = 20$ dB, $\gamma_{12} = 20$ dB.

7.2.3 With direct link between R_1 and R_2

Figures 7.6, 7.7 show the comparison of rate regions achieved using the alternating two path relay scheme(AR-DF) with the outer bound for two cases : 1)$\gamma_{a1} = 10$ dB, $\gamma_{b1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b2} = 10$ dB, $\gamma_{12} = 15$ dB and 2)$\gamma_{a1} = 20$ dB, $\gamma_{b1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b2} = 20$ dB, $\gamma_{12} = 20$ dB. The CF-CMAC and convex hull are also shown. Case 1 is shown to achieve the protocol specific upper bound in [14]. We see that the AR-DF performs best at the axes, while compute and forward is still the best strategy for achieving symmetric rates.

7.3 Comparison of Outer bounds

In this section, we plot and compare the different outer bounds derived in Chapter 5. The outer bounds are computed for two channel conditions: Case I: $\gamma_{a1} = 10$ dB, $\gamma_{b1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b2} = 10$ dB, $\gamma_{ab} = 7$ dB, $\gamma_{12} = 7$ dB, Case II: $\gamma_{a1} = 14$ dB, $\gamma_{b1} = 5$ dB, $\gamma_{a2} = 6$ dB, $\gamma_{b2} = 12$ dB, $\gamma_{ab} = 8$ dB, $\gamma_{12} = 8$ dB.

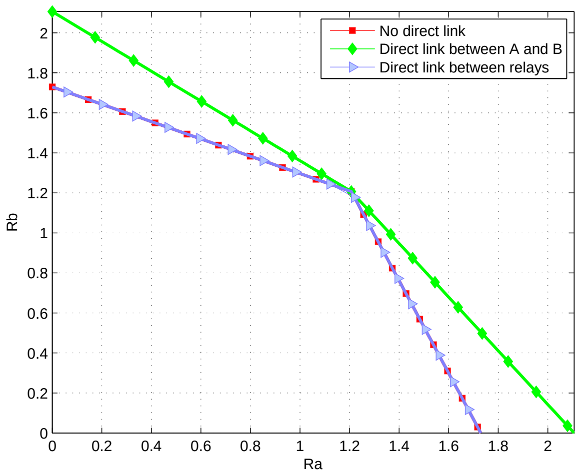

Figure 7.8: Outer bounds for capacity region of the gaussian diamond channel: $\gamma_{a1} = 10$ dB, $\gamma_{b1} = 10$ dB, $\gamma_{a2} = 10$ dB, $\gamma_{b2} = 10$ dB, $\gamma_{ab} = 7$ dB, $\gamma_{12} = 7$ dB.

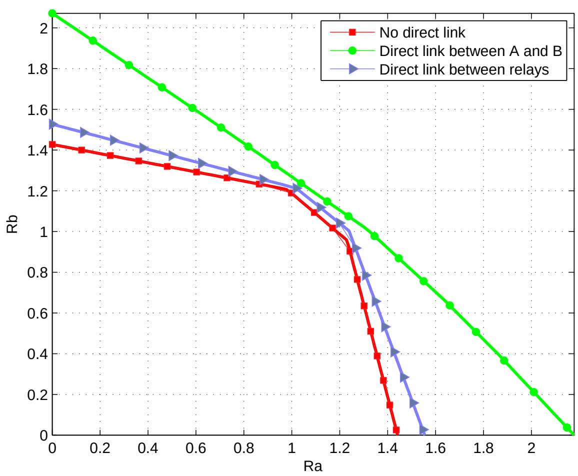

Figure 7.9: Outer bounds for capacity region of the gaussian diamond channel: $\gamma_{a1} = 14$ dB, $\gamma_{b1} = 5$ dB, $\gamma_{a2} = 6$ dB, $\gamma_{b2} = 12$ dB, $\gamma_{b2} = 8$ dB, $\gamma_{b2} = 8$ dB.

Figure 7.8 shows the outer bounds for Case I. The link between the nodes A and B provides a direct path of communication between them, so this outer bound is better than the one without direct link in all channel conditions. The link between the relays does not provide any improvement for the symmetric channel case. Figure 7.9 shows the outer bounds for Case II. The direct link between relays is observed to increase the achievable region only when the three hop path (through the $R_1 - R_2$ link) is better than the other two in terms of SNR. In other cases, this bound is found to overlap with the outer bound for the diamond channel without direct links.

CHAPTER 8

CONCLUSION

8.1 Contribution of this thesis

The Gaussian two-way relay channel with direct link and the Gaussian two-way diamond channel are studied. An analytical outer bound for the capacity region of the gaussian two way relay channel has been derived. This analytical outer bound is found to be close to the numerical outer bound in most cases, and it helps in analysing the outer bound better.

Generalised outer bounds for the capacity region of the Gaussian Diamond Relay channel with and without direct links have been derived. Existing protocols for communication through a diamond relay channel without direct link have been studied. Three new protocols- 2-way MDF, CF-CMAC and CF-BC have also been proposed, that use different subsets of states of a diamond relay network. The 2-way MDF protocol, which is an extension of the existing MDF protocol for one-way relaying in a diamond channel, is found to achieve capacity for some channel conditions. The CF-CMAC protocol, based on compute and forward technique is found to achieve rate rate pairs that the 2-way MDF cannot, in several channel conditions. CF-CMAC protocol is also found to be better than CF-BC because of the Compound MAC state. The achievable rate regions of all the protocols are compared with the outer bound. While the two-way MDF protocol is found to be close to capacity near the axes, the compute and forward protocols achieve better rates along the $45°$ line. A convex combination of the 2-way MDF and CF-CMAC protocols is found to be close to the outer bound. Achievable rate regions for the diamond channel with direct links have also been investigated.

8.2 Future Work

New protocols for communication over a diamond channel with direct links can be investigated, since there is scope for improving the achievable rate regions obtained in this thesis. Also, the 2X2 interference network of State 14 of Figure 3.2 can be exploited to improve the achievable rate region. Inner and outer bounds for the full-duplex diamond channel can also be investigated.

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LIST OF PAPERS BASED ON THESIS

1. Prathyusha V,Srikrishna Bhashyam and Andrew Thangaraj, "The Gaussian Two-way Diamond Channel", submitted to Globecom 2013